### A Color Palette based Interface for Biological Iridescences Rendering

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#### **Abstract**

Iridescence is an optical phenomenon of surfaces in which hue changes in proportion to the angles of observation and illumination. Iridescent colors can be observed on many biological objects including morpho butterflies and Japanese jewel beetles. While ordinary colors are induced by pigments in surface, the iridescent colors are caused by interference of light due to microstructure covering the objects. To render iridescences in computer graphics, users need to consider a lot of physical parameters such as index of refraction or film layer thickness. However, in such input systems, users are difficult to predict rendering results since iridescence is affected by combination of those many physical factors. In this paper, we propose a color palette based interface for biological iridescences rendering, with which users can pass information for rendering iridescences to system by picking colors instead of by inputting physical data of surface. Incident angle dependency of reflectance spectrum is studied in details to achieve physically sound simulation and color picking user interface. We also evaluated our system through rendering results(of morpho butterflies and Japanese jewel beetles) and user study in the latter half of this article.

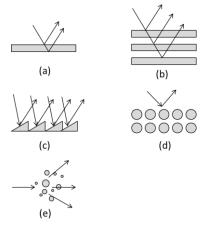
**Keywords:** iridescence color, spectral rendering, user interface

#### 1 Introduction

Iridescence is an optical phenomenon of surfaces in which hue changes in proportion to the angles of observation and illumination [1]. We can easily find iridescence colors on objects such as compact discs, bubbles and also on animals such as morpho butterflies and Japanese jewel beetles. Iridescences are caused by physical phenomena including thin film interference, multilayer interference, diffraction gratings, photonic crystals and light scattering [2] (Figure 1). Among those mechanisms, iridescence treated in our system is limited to the

one due to biological multilayer systems which are found in morpho butterflies, Japanese jewel beetles, neon tetra and many other animals (Figure 2).

Generally, to render iridescences in computer graphics, users need to consider a lot of physical parameters of surface such as index of refraction or film layer thickness. However, in such input systems, users are difficult to predict rendering results since iridescence is affected by combination of those many physical factors.



**Figure 1** Various optical phenomena causing iridescences (a)-(e) are thin film interference, multilayer interference, diffraction gratings, photonic crystals and light scattering respectively.

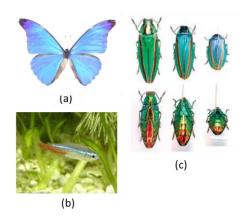
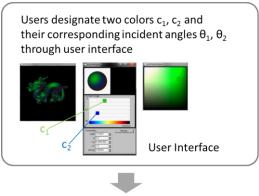
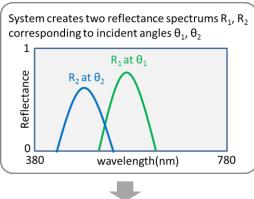
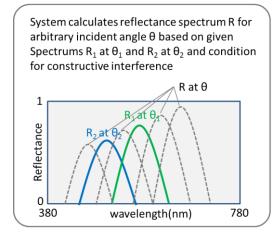


Figure 2 Examples of iridescences caused by multilayer structures

(a)-(c) are morpho butterfly(M. rhetenor), neon tetra and Japanese jewel beetle respectively[3,4].







**Figure 3** The process to calculate reflectance spectrums for iridescences rendering from user inputs

Once again, the characteristics of iridescence color is laid on its color changes in proportion to the incident angles of light. Thus, the aim of this article is to achieve a system that calculates correspondence between reflectance spectrum( relationship between reflectance spectrum and color is described in Appendix) and its incident angle with enhanced usability. We propose a user interface that allows users to input information for rendering iridescences by designating colors and their incident angles instead of by considering physical parameters. From the information given by users, two reflectance spectrums for corresponding incident angles are defined in our system. Then, correspondence between reflectance spectrums and incident angles is calculated based on the two

spectrums and equation of constructive interference, which leads to full information for iridescence color rendering. These processes are summarized in Figure 3.

Section 2 reviews previous works on iridescence rendering in computer graphics. Section 3 describes methods to model reflectance spectrums and calculates correspondence between the spectrums and incident angles. Section 4 presents user interface and section 5 describes evaluation on our system through reflectance spectrum comparison, rendering results and user study. Section 6 concludes this article and discusses the limitation of our methods.

#### 2 Related Work

Several methods have been developed to represent iridescence phenomenon. Hirayama et al.[5] proposed an rendering model for iridescent colors appearing on natural objects through the use of a hypertexture. Hirayama et al.[6] also developed an accurate illumination model for objects coated with multilayer films considering composite reflectance and transmittance. Stam[7] proposed a new class of reflection models for metallic surfaces that handle the effects of diffraction. Gondek et al.[8] developed a method that represents interference of light due to paint and thin film by using a Monte Carlo ray tracer to cast rays into a surface. Iwasawa et al.[9] proposed rendering methods for models with complicated micro structures considering diffraction, light scattering and anisotropy. Nagata et al.[10] developed a rendering model for pearls by calculating multiple reflection in spherical bodies. Sun [11] proposed a model that handles rendering of biological objects by analytical calculation.

As a whole, previous studies have focused on how to simulate iridescences realistically and with these efforts this beautiful phenomenon can be reproduced close to real one by computer graphics nowadays. On the other hand, there is no research that handles user interfaces that assist users to input rendering information to system intuitively in this field.

#### 3 Iridescence Model

In this section, we discuss how to model reflectance spectrums of biological multilayer systems and how to calculate correspondence between reflectance spectrums and incident angles. The method to designate colors(or spectrums) and their corresponding incident angles will be treated in section 4 for easy understanding of this article.

# 3.1 Reflectance Spectrum of Biological Multilayer System

In multilayer systems(figure 4), the condition for constructive interference is expressed as[2]

$$2(n_A d_A \cos \theta_A + n_B d_B \cos \theta_B) = m\lambda \tag{3.1.1}$$

where  $n_A$ ,  $n_B$ ,  $d_A$ ,  $d_B$ ,  $\theta_A$ ,  $\theta_B$  refer to index of refraction, layer thickness and refractive angle of layer A and B respectively  $\lambda$  is wavelength of incident ray and m is integer greater than 0. In most cases, multilayer system of biological objects satisfy equation (3.1.1) with the range of  $380 < \lambda < 780$  (namely, the range of visible wavelength of light) when m equals 1[12].

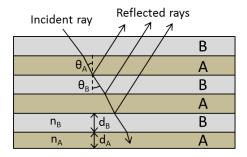
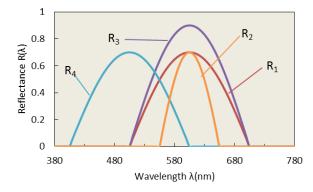


Figure 4 Multilayer system



**Figure 5** Shapes of reflectance spectrum based on equation (3.1.2)

The values of ( $\lambda_{peak}$ , w, h) for spectrum  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$  are (600, 200, 0.7), (600, 100, 0.7), (600, 200, 0.9), (500, 200, 0.7), respectively.

This implies that reflectance spectrum shows one peak in visible wavelength region, therefore, we define the shape of reflectance spectrum R as following expression:

$$R(\lambda_{peak}, w, h, \lambda) = h \cdot \cos((n/w)(\lambda - \lambda_{peak}))$$

$$(if \lambda_{peak} - w/2 < \lambda < \lambda_{peak} + w/2)$$

$$R(\lambda_{peak}, w, h, \lambda) = 0$$

$$(if \lambda < \lambda_{peak} - w/2 \text{ or } \lambda_{peak} + w/2 < \lambda)$$

$$(3.1.2)$$

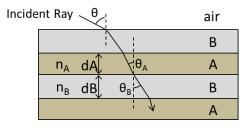
where w, h,  $\lambda_{peak}$  are width, height and peak wavelength of spectrum respectively. These 3 factors define the shape of reflectance spectrum as shown in Figure 5 and are provided through color palette which will be discussed in section 4 in detail.

## 3.2 Correspondence between Reflectance Spectrum and Incident Angle

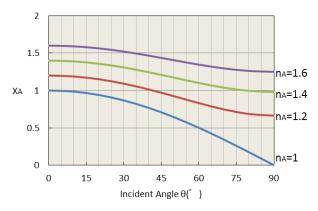
Reflectance spectrum R assumed in section 3.1 doesn't include incident angle terms that are necessary to explain iridescence's behavior. Therefore, we define correspondence between reflectance spectrum and incident angle here. In other words,  $\lambda_{peak}$ , w, h of equation (3.1.2) will be described in terms of incident angle in this section.

From the discussion in section 3.1, the condition for constructive interference in biological multilayer system is expressed as

$$\lambda_{peak} = 2(n_A d_A \cos \theta_A + n_B d_B \cos \theta_B) \tag{3.2.1}$$

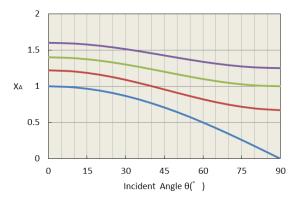


**Figure 6** Multilayer structure above which air layer exists  $\theta$  is incident angle from air layer.



**Figure 7** Curves of  $X_A$  for different values of  $n_A$  between 1 and 2

 $n_A$  values of the curves are 1, 1.2, 1.4, 1.6 respectively from bottom to top.



**Figure 8** Curves of  $X_A$  constructed from linear combination of  $\cos^2\theta$ ,  $\cos\theta$  and a constant.

(l, m, n) values of the curves are (0, 1, 0), (0.5, 0.05, 0.67), (0.4, 0, 1), (0.35, 0, 1.25) respectively from bottom to top.

It is natural to assume that the region above multilayer is air as shown in Figure 6, and in such case, the following relationship (3.2.2) is derived according to Snell's Law[13],.

$$n_A \sin \theta_A = n_B \sin \theta_B = 1 \cdot \sin \theta$$
 (3.2.2)

where  $\theta$  is incident angle from air layer. Given equations (3.2.1) and (3.2.2), then

$$\lambda_{peak} = 2(d_A X_A + d_B X_B)$$

$$(X_A = (n_A^2 - \sin^2 \theta)^{1/2}, X_B = (n_B^2 - \sin^2 \theta)^{1/2})$$
(3.2.3)

In figure 7, we plot the curves of  $X_A$  for different values of  $n_A$  between 1 and 2. The curve is cosine when  $n_A$  equals 1, but for other values of  $n_A$ , curves are S-shaped. To simplify  $X_A$  that is expressed with root term, we assume  $X_A$  as linear combination of  $\cos^2\theta$ ,  $\cos\theta$  and a constant:

$$X_A = l\cos^2\theta + m\cos\theta + n \tag{3.2.4}$$

where l, m are coefficients and n is .a constant. In figure 8, the curves of  $X_A$  of equation (3.2.4) are plotted with (l, m, n) values, (0, 1, 0), (0.5, 0.05, 0.67), (0.4, 0, 1), (0.35, 0, 1.25) respectively from bottom to top from the plotted curves, it is proved that  $X_A$  of new form is sufficiently similar to the original equation by selecting appropriate values of l, m, n.

Since  $X_B$  can be also described by equation (3.2.4), applying  $X_A$ ,  $X_B$  of new forms to equation (3.2.3), the following equation (3.2.5) is derived:

$$\lambda_{peak} = k(p\cos^2\theta + (1-p)\cos\theta) + q \tag{3.2.5}$$

where k, p are coefficients and q is a constant. Coefficient  $p(0\sim1)$  is a parameter that determines the shape of  $\lambda_{peak}(\theta)$ between cosine shape(p=0) and s shape(p=1) and this will be inputted by users.

Given two pairs of peak wavelength and its corresponding incident angle, namely,  $(\lambda_{peak1}, \theta_1)$ ,  $(\lambda_{peak2}, \theta_2)$ , the following relationships are derived from equation (3.2.5):

$$\lambda_{peakl} = kY_l + q(Y_l = p\cos^2\theta_l + (1-p)\cos\theta_l)$$
 (3.2.6)

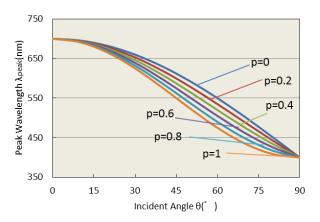
$$\lambda_{peak2} = kY_2 + q(Y_2 = p\cos^2\theta_2 + (1-p)\cos\theta_2)$$
 (3.2.7)

Solving equation (3.2.6) and (3.2.7), then

$$k = (\lambda_{peak1} - \lambda_{peak2})/(Y_1 - Y_2)$$
(3.2.8)

$$q = (\lambda_{peak1} Y_2 - \lambda_{peak2} Y_1) / (Y_2 - Y_1)$$
 (3.2.9)

Applying equation (3.2.8) and (3.2.9) to equation (3.2.5), the final relationship between incident angle  $\theta$  and corresponding peak wavelength  $\lambda_{peak}$  is derived (note Figure 9).



**Figure 9** Relationship between peak wavelength  $\lambda_{peak}$  and corresponding incident angle  $\theta$ , given (700, 0) and (400, 90) for values of  $(\lambda_{peak1}, \theta_1)$  and  $(\lambda_{peak2}, \theta_2)$ p values of the curves are 1, 0.8, 0.6, 0.4, 0.2, 0 respectively from bottom to top. As incident angle increases, peak wavelength decreases, which is general characteristics of reflectance spectrum caused by multilayer structures.

For the remaining factors of equation (3.1.2), w, h, we empirically define that those are linear functions of  $\lambda_{peak}$ .

From the discussion above, given reflectance spectrum  $R_1(=R(\lambda_{peak1}, w_1, h_1, \lambda))$  and  $R_2(=R(\lambda_{peak2}, w_2, h_2, \lambda))$  corresponding to incident angle  $\theta_1$ ,  $\theta_2$  respectively, reflectance R for arbitrary incident angle  $\theta$  is expressed as

$$R=R(\lambda_{peak}, w, h, \lambda)$$
 (3.2.10)

$$h = (\lambda_{peak} - \lambda_{peakl})(h_2 - h_1)/(\lambda_{peak2} - \lambda_{peakl}) + h_1$$
 (3.2.11)

$$w = (\lambda_{peak} - \lambda_{peak1})(w_2 - w_1) / (\lambda_{peak2} - \lambda_{peak1}) + w_1$$
(3.2.12)

 $w = (\lambda_{peak} - \lambda_{peakI})(w_2 - w_I)/(\lambda_{peak2} - \lambda_{peakI}) + w_I$  $\lambda_{peak} = k(p\cos^2\theta + (1-p)\cos\theta) + q$ (3.2.13)

k, q in equation (3.2.13) can be calculated from equation (3.2.8) and (3.2.9). Note that now h, w and  $\lambda_{peak}$  are all functions of incident angle  $\theta$ .

#### 4 User Interface

User interface consists of 3 parts, output window, parameter setting window, color palette window as shown in Figure 10. In this section, we first look into each part of user interface and then describe how to use to render iridescence colors.

#### 4.1 Output window(Figure 10(a))

Rendering results are drawn here based on the information from parameter setting window and color palette.

#### 4.2 Parameter setting window(Figure 10(b))

In this part, rendering parameters for iridescences such as the value of p( which doesn't affect rendering result so much) and two pairs of peak wavelength and its corresponding incident angle,  $(\lambda_{peak1}, \theta_1)$  and  $(\lambda_{peak2}, \theta_2)$  are designated by users. The region where these peak wavelengths and incident angles are inputted is  $\lambda_{peak}$ - $\theta$  plane(Figure 11). In the plane, x-axis(380~780) and y-axis(90~0) represents incident angle  $\theta$ and peak wavelength  $\lambda_{peak}$ , respectively. There are two mova-

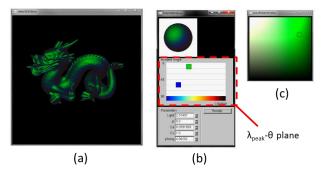


Figure 10 User interface

(a)-(c) are output window, parameter setting window and color palette window respectively.

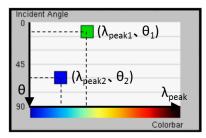


Figure 11  $\lambda_{peak}$ - $\theta$  plane

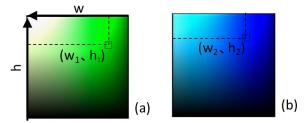


Figure 12 Color palette

(a),(b) are palettes when squares of  $(\lambda_{peak1}, \theta_1)$  and  $(\lambda_{peak2}, \theta_2)$  are selected on  $\lambda_{peak}$ - $\theta$  plane respectively.

ble squares, and their colors are calculated from reflectance spectrum  $R_1$  and  $R_2$ ( Note Appendix). Based on the coordinates of the squares, the values of  $(\lambda_{peak1}, \theta_1)$  for  $R_1$  and  $(\lambda_{peak2}, \theta_2)$  for  $R_2$  are determined

#### 4.3 Color palette window(Figure 10(c))

In this part, users set spectrum height  $h_i$  and spectrum width  $w_i$  for  $R_i$  when given  $\lambda_{peaki}$  and  $\theta_i$  (i=1, 2)(Figure 12). In color palette, x-axis(0~1) and y-axis(400~40) represent w and h respectively. The color of coordinate (w, h) in the palette corresponds to color of spectrum  $R_i(\lambda_{peaki}, w_i, h_i, \lambda)$ . Therefore, right side of the palette corresponds to spectrum with narrow width (Note that saturation is high and brightness is low in right side of the palette). The upper side of the palette refers to spectrum whose reflectance is high (Note that upper region of the palette is bright). There is one controllable square, and with its coordinate the values of  $w_i$  and  $h_i$  are determined.

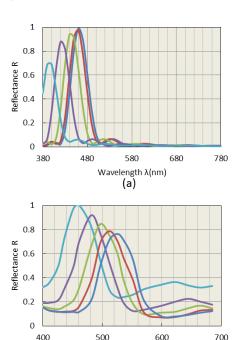
#### 4.4 User input flow

- 1. First, users designate hue of the color and its incident angle of purpose by moving one of the squares on  $\lambda_{\text{peak}}$ - $\theta$  plane in parameter setting window. As a result, peak wavelength  $\lambda_{\text{peak}I}$ , incident angle  $\theta_I$  for reflectance spectrum  $R_I$  is determined.
- 2. After that, users control brightness and saturation by moving a square on color palette, which determines height  $h_I$  and width  $w_I$  for  $R_I$ .
- 3.  $\lambda_{peak2}$ ,  $h_2$ ,  $w_2$  for  $R_2$  and  $\theta_2$  are determined by repeating flow 1, 2 above again with the other square on  $\lambda_{peak}$ - $\theta$  plane.
- 4. Based on user input above,  $R(\lambda_{peak}, w, h, \lambda)$  is calculated for arbitrary incident angle  $\theta$  in the system.

#### 5 Evaluation

#### 5.1 Reflectance Spectrum Comparison

Reflectance spectrum R for arbitrary incident angle  $\theta$  is calculated based on user inputted spectrums  $R_1$  and  $R_2$  in our system as discussed in section 3.2, Here, the calculated correspondence between R and  $\theta$  by our method is compared with published reflectance spectrum data of morpho butterflies and Japanese jewel beetle for validation.

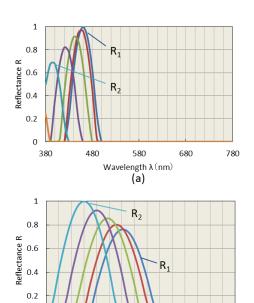


**Figure 13** Reflectance spectrum and its incident angle dependency( from reference data[11,14])

(a)-reflectance spectrum of morpho butterfly. From right to left, spectrums correspond to incident angles  $0^{\circ}$ ,  $10^{\circ}$ ,  $20^{\circ}$ ,  $30^{\circ}$ ,  $40^{\circ}$  respectively.

Wavelength λ(nm) (b)

(b)-reflectance spectrum of Japanese jewel beetle. From right to left, spectrums correspond to incident angles 20°, 30°, 40°, 50°, 60° respectively.



**Figure 14** Reflectance spectrum and its incident angle dependency( calculated by our method)

Wavelength λ(nm)

700

400

(a)-reflectance spectrum of morpho butterfly. From right to left, spectrums correspond to incident angles  $0^{\circ}$ ,  $10^{\circ}$ ,  $20^{\circ}$ ,  $30^{\circ}$ ,  $40^{\circ}$  respectively.  $R_1$  and  $R_2$  are selected as  $R(460, 70, 1, \lambda)$  at incident angle  $0^{\circ}$  and  $R(395, 60, 0.7, \lambda)$  at incident angle  $40^{\circ}$ . (b)-reflectance spectrum of Japanese jewel beetle. From right to left, spectrums correspond to incident angles  $20^{\circ}$ ,  $30^{\circ}$ ,  $40^{\circ}$ ,  $50^{\circ}$ ,  $60^{\circ}$  respectively.  $R_1$  and  $R_2$  are selected as  $R(530, 110, 0.73, \lambda)$  at incident angle  $10^{\circ}$  and  $R(460, 110, 1, \lambda)$  at incident angle  $60^{\circ}$ .

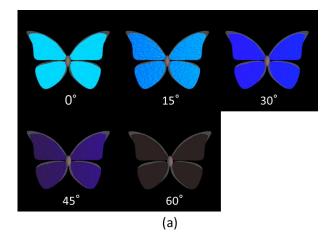
The reference data are described in Figure 13. The data of morpho butterflies is derived from Sun's physical simulation[11]. The other data is reflectance spectrum of Japanese jewel beetle measured by Stavenga[14]. Spectrum shift in proportion to incident angle is confirmed.

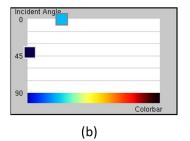
Incident angle dependency of reflectance spectrum is calculated by our method in Figure 14. The values ( $\lambda_{peaki}$ ,  $w_i$ ,  $h_i$ ) of  $R_i$  and its corresponding  $\theta_i$  (i=1,2) for calculation are (460, 70, 1) at 0°, (395, 60, 0.7) at 40° for morpho butterflies and (530, 110, 0.73) at 10°, (460, 110, 1) at 60° for Japanese jewel beetles. These  $R_I$  and  $R_2$  are measured from reference data in Figure 13.

From the comparison of our results and reference data, it can be said that angular dependency of spectrum by our method is sufficiently in accordance with the one by other groups.

#### **5.2 Rendering Results**

Rendering results of morpho butterfly and Japanese jewel beetle are depicted in Figure 15 and Figure 16. The values of  $R_1$  and  $R_2$  discussed in section 5.1 are used to render each object. The rendered images show color shifts (from cyan to violet for morpho butterfly and from green to violet for Japa-





**Figure 15** Rendering results of morpho butterfly (a)-incident angles are described below each image. (b)- $\lambda_{\text{peak}}$ - $\theta$  plane setting for the rendering.

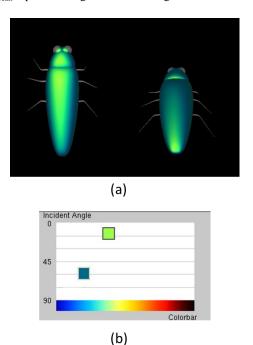


Figure 16 Rendering results of Japanese jewel beetle (a)-rendering images of two different position. (b)- λ<sub>peak</sub>-θ plane setting for the rendering.

nese jewel beetle) that agree with the observed iridescences of them.

#### 5.3 User Study

We conducted user study to compare usability of our system and the one of physical parameters input based system through measuring time to render given sample images of iridescences. The latter system was implemented based on Sun's empirical model[11] for biological iridescences due to multilayer structure. In this system, to render iridescences, users are needed to input physical properties of surface such as thickness of film, thickness of air, index of refraction of film and one auxiliary parameter.

The protocol used in this study was as following.

- 1. 3 sample images, image 1, 2, 3(Figure 17) of iridescence are given to participants.
- 2. The Participants are required to render same images of the given samples using two systems above. Specifically, image 1, 2, 3 are rendered with one system in sequence, and then, these are rendered again with the other system. Time to get the same image was measured for each image.

8 participants (who are not familiar with iridescence phenomenon) were divided into 2 groups, A and B for this test. Participants of A group used our system first and the system based on Sun's model later in part 2 of the protocol. Participants of B group tested systems in opposite order. The reason to divide two groups is to consider the influence of the order testing systems.

The result of user study is shown in Figure 18. In Figure 18, y axis means average time for each group A( represented as (A)) or B( represented as (B)) to get images using system based on Sun's model( represented as Sun) or our system( represented as Park). X axis represents sample image number. From the results, it is obvious that the order of using system doesn't influence results in that A group and B group shows similar results for the same system. In addition, it can be said that our color palette based user interface provides intuitive usability since average time to get images by our system are shorter than the one by the other system.







**Figure 17** 3 sample images of iridescences From left to right, samples are image1, image2 and image3 respectively.

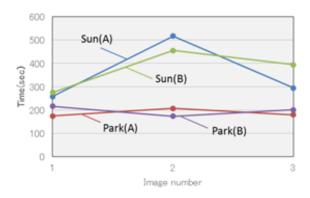


Figure 18 Results of user study

#### 6 Conclusion

In this paper, we proposed a color palette based user interface for biological iridescences rendering.

This system allows users to pick two colors from color palette corresponding to their incident angles. From this information, two reflectance spectrums are defined and correspondence between reflectance spectrums and incident angles is calculated based on the two spectrums and equation of constructive interference.

We also evaluated our system in three points, reflectance spectrum validation, rendering results and user study. In reflectance spectrum comparison and rendering results, calculated spectrums and rendered images of morpho butterflies and Japanese jewel beetles based on our method matched well with both real spectrum data and real images. In user study, users rendered sample images with our new system and previous system(namely, physical parameters input based system), and it turned out that users can render aimed iridescences with shorter time when they used our system.

Even though our approach provides not only effective usability, but also physically sound rendering results, the method adopted in this paper is only for iridescences caused by biological multilayer system. As mentioned in section 1, since iridescences can occur through various mechanisms, further research should be carried out to achieve general usability covering such iridescences.

### Appendix

#### Transformation of Spectrum to RGB

By using color matching function  $x(\lambda)$ ,  $y(\lambda)$ ,  $z(\lambda)$ (which is defined by the International Commission on Illumination(CIE)) and following equation(A.1), reflectance spectrum  $R(\lambda)$  can be transformed to 3 standard primaries, X, Y and Z.

$$X=k \int R (\lambda)x(\lambda)d\lambda$$

$$Y=k \int R (\lambda)y(\lambda)d\lambda$$

$$Z=k \int R (\lambda)z(\lambda)d\lambda$$
(A.1)

Integral range is from 380 to 780 and k is coefficient. Transforming XYZ to RGB is represented as equation(A.2)[15].

$$R=3.2406X-1.5372Y-0.4986Z$$
  
 $G=-0.9689X+1.8758Y+0.0415Z$  (A.2)  
 $B=0.0557X-0.2040Y+1.0570Z$ 

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