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Impossible figures' morphological recognition and analysis of inconsistent rectangles

A basic study for digital works relating to impossible figures

Tsuruno,Sachiko
Kindai University,Kyushu University
sachiko@fuk.kindai.ac.jp

Tomimatsu,Kiyoshi
Kyushu University
tomimatu@design.kyushu-u.ac.jp

Abstract
Impossible figures are known to be motifs of the Dutch artist M. C. Escher's lithographs. However, impossible figures cannot be strictly defined geometrically because they are mental images of solid objects. In other words, viewers perceive two-dimensional (2D) drawings as three-dimensional (3D) structures, although these structures cannot be realized in 3D space. Regardless of the mental images, viewers' morphological different recognition of impossible figures have not been sufficiently researched; thus, we performed two experiments to address this gap. In the first experiment, the participants observed each sample figure individually in random order and then stated whether, according to them, it was an impossible or possible figure. Approximately half the participants labeled some sample figures as possible figures in spite of them being impossible geometrically. The results indicated that perceptions of impossible figures differ according to the individual and the figures themselves. We also obtained widely differing results between four inconsistent rectangles that had the external contours of possible rectangles. To address this variability, we focused on the inconsistent rectangles in the second experiment. The four rectangles were sub-classified into 28 categories, and the participants were asked whether each of the 28 figures was impossible or possible, similar to the procedure followed in the first experiment. The sub-classified rectangles were broken down into polygons to analyze the results. Finally, we extracted an element that led to participants' perception of possible figures and two elements that led to their perception of impossible figures.

Keywords:impossible figure, perception, inconsistent rectangle

1 Introduction

Artwork containing impossible figures can be traced to the 16th century; however, some of the major works were created after Reutersvard's[1] 1934 artwork depicting an impossible tribar comprising nine cubes. M.C. Escher's lithographs[2], created around 1960, used impossible figures as motifs and these works are very well known. Such figures have been studied in some fields. R. and L. Penrose[3] and Gregory [4] described visual perception mechanisms of impossible objects. Robinson[5],Draper [6], Cowan[7][8][9], Kulpa[10][11], Gillam[12],Young et al.[13], and Shepard[14] also studied impossible figures psychologically while Ernst[2][15] structurally explained impossible figures. Sugihara[16][17] formulated the algebraic structure of a 3D polyhedron's degrees of freedom, which was projected onto a 2D screen as a congruent figure. T´erouanne[18] and Uribe[19] also researched impossible figures in the field of mathematics. Huffman[20], Clowes[21], Tsuruno[22][23], Savransky et al.[24],Owada and Fujiki[25], Wu et al.[26], and Elber[27] approached impossible figures from the computer science and graphics perspectives. Furthermore, a lot of creative works on the impossible figure motif have been published by many creators including Del-Prete[1], Mey[1], Fukuda[1], Hamaekers[1], Yturralde[28], Sugihara[29], and Tsuruno[30].

2 Research objectives and method

Impossible figures are studied from the various fields, as described above. However, the figures themselves cannot be strictly defined geometrically because they are mental images of solid objects. That is, viewers perceive two-dimensional (2D) drawings as three-dimensional (3D) structures, although these structures cannot be realized in 3D space. Viewers are attracted to this contradiction and feel marvelous. Even if a figure is geometrically impossible, viewers find it less attractive if they cannot easily recognize it as an impossible figure. In fact, previous studies appear to include some figures that many viewers cannot easily interpret as impossible figures. Furthermore, when we published several works that used impossible figures as motifs, there were always some viewers who did not recognize these as impossible figures. Since impossible figures are mental images, differing perceptions are assumed to emerge according to different individuals and the figures themselves. Cole et al.[31] and Lee et al.[32] examined the perception of 3D (possible) figures from line drawing figures; however, as far as we know, no study has investigated the different perceptions of impossible figures. Therefore, in this paper, we study the different perceptions of impossible figures.

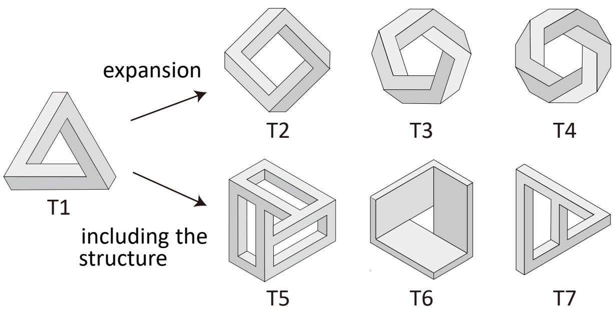


Figure 1 Penrose triangle

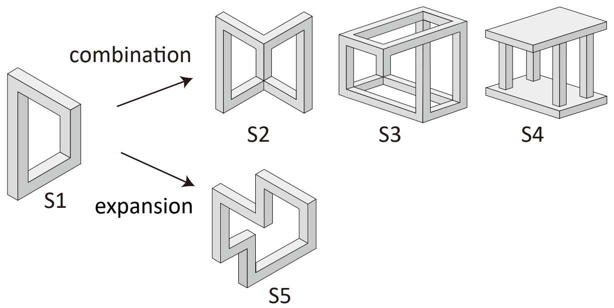


Figure 2 Skew trapezoid

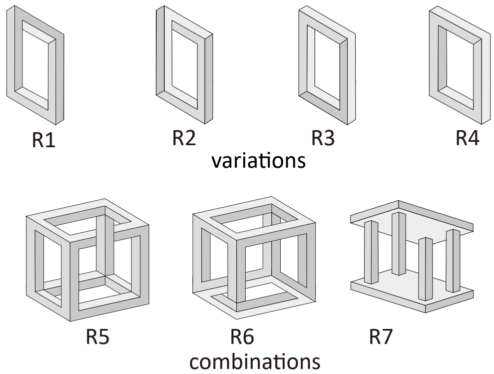


Figure 3 Inconsistent rectangle

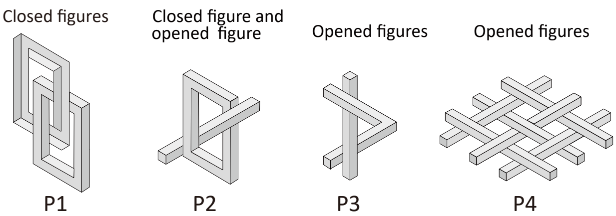


Figure 4 Possible figures inconsistently placed

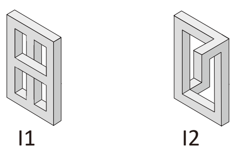


Figure 5 Inconsistent internal connection

Thus, we conducted two experiments. In Experiment 1, we prepared categorized sample figures and investigated different perceptions depending on individuals and the morphology of the impossible figures. We then focused on inconsistent rectangles that obtained different results despite having similar forms in Experiment 1. Experiment 2 was performed to examine the cause of these results.

3 Impossible figure perception experiment (Experiment1)

3-1 Sample figures with inconsistent depth

We prepared a set of 25 sample figures that were classified according to their morphological attributes. Five categories were created, as shown in Figures 1–5. Figure 1 shows a Penrose triangle group with T1 as a Penrose triangle itself and T2–T4 as figures expanded from a Penrose triangle and composed of twisted corner repetitions. T5–T7 include Penrose triangles in their structures. Figure 2 is a skew trapezoid group. S1 is a skew trapezoid itself, S2–S4 are combinations of skew trapezoids, and S5 is an expansion. Figure 3 indicates a group of inconsistent rectangles having the external contours of possible rectangles. R1–R4 are variations of the inconsistent rectangle and R5–R7 are combinations. Figure 4 shows a group of inconsistently placed possible figures. Each of the figures in P1–P4 comprises inconsistently placed and disconnected possible figures. In Figure 5, I1 and I2 are rectangles that are internally constructed of inconsistent connection. These 25 figures were drawn with geometrically inconsistent depth in 3D space under the presupposition that the polygons indicate plane surfaces and the figures are composed of convex parts. Further, five possible figures in Figure 6 were provided as dummy figures.

3-2 Experiment 1

This experiment was performed to investigate whether differing perceptions are observed according to individuals and the morphology of the impossible figures. Fifty-eight participants (46 male, 12 female, average age 22 years) took part in the experiment. They observed the figures while seated at classroom desks lit by lamps of 300 lx or more. Each sample figure was printed on the left side of a 148 mm x 210 mm sheet, and the participant marked his/her answer on the right side of the sheet. The figures were drawn using black 0.5pt lines, and each polygon was slightly shaded in monochrome. We did not fix the viewing time for each figure to enable participants to take their time when interpreting the figure. However, according to the execution result, all participants finished marking the check boxes for all 30 figures within 15 minutes. To decrease the influence of presentation order on the results, the sheets were shown in random order; that is, each participant observed them in a different order. Two explanations were given in advance:

1) Every figure is composed of convex parts.

2) Possible figures can exist as spatial objects that can be observed from multiple viewpoints in 3D space. Thus, even if the figure on the sheet corresponds to a spatial object only from a specific viewpoint, it is not a possible figure.

3-3 Result

The results from Experiment 1 are shown in Table 1. It denotes

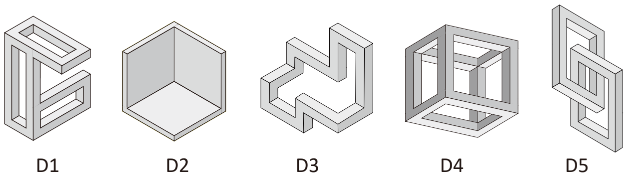


Figure 6 Dummy figures (possible figures)

sign	figure	Ratio of Possible	sign	figure	Ratio of Possible	sign	figure	Ratio of Possible
S4		55%	T2		21%	S3		3%
T4		50%	P2		19%	R2		3%
R4		41%	S2		17%	R1		2%
T3		38%	R8		17%	R6		2%
R3		33%	T5		14%	R5		0%
P1		33%	S1		12%	D1		98%
P3		29%	I2		10%	D2		95%
T6		28%	T1		9%	D3		98%
P4		24%	I1		5%	D4		95%
S5		22%	T7		3%	D5		83%

Table 1 Result of Experiment 1

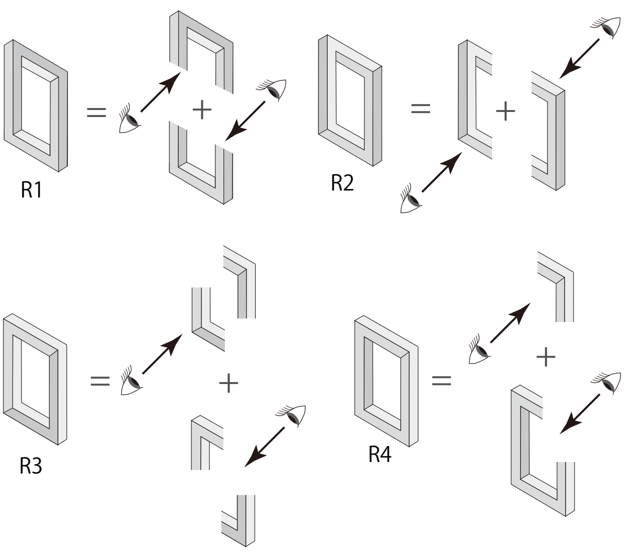


Figure 7 Breaking down into possible parts

the decreasing order of the ratio of participants who answered "Possible." This result demonstrated that figures interpreted as "possible figures" depended on each individual participant, even if it was a geometrically impossible figure. The results for R1–R4 were of particular interest. Although R4 was similar to R1, R2, and R3, the ratio of participants who answered "Possible" varied; R4 (41%) greatly differed from R1 (2%) and R2 (3%). Given this vast difference, we investigated inconsistent rectangles in Experiment 2.

4 Inconsistent rectangle perception experiment (Experiment 2)

4-1 Hypothesis

Inconsistent rectangles R1-4 have different ways of connecting with each other's corners. Each figure can be broken down into possible parts, as shown in Figure 7. Thus, R1 can be divided into the upper figure, viewed from below, and the lower figure, viewed from above; hereafter, such an inconsistent rectangle is termed UD-type. R2 can be divided into the right figure, viewed from above, and the left figure, viewed from below, hereafter termed RL-type. R3 can be divided into two diagonal pair of corners, where each corner pair is a part of a possible rectangle. The upper right and lower left corners are viewed from below while the upper left and lower right corners are viewed from above. Such an inconsistent rectangle is termed DG-type. R4 includes only the top right corner, drawn from a lower viewpoint; the other three corners are drawn from an upper viewpoint. Such an inconsistent rectangle is termed C-type. We then built the following hypotheses:

1) An inconsistent UD-type rectangle has a high possibility to be perceived as an impossible figure.

2) An inconsistent RL-type rectangle has a high possibility to be perceived as an impossible figure.

3) An inconsistent DG-type rectangle has a possibility not to be perceived as an impossible figure.

4) An inconsistent C-type rectangle has a possibility not to be perceived as an impossible figure.

4-2 Inconsistent rectangle sub-classification

As shown in Figure 8, the four possible rectangles are provided as dummy figures. P-Dv and P-Dh are viewed from above, and P-Uv and P-Uh are viewed from below. Furthermore, P-Dv and P-Uv are vertically long type of rectangles while P-Dh and P-Uh are the horizontally long type, which are used to examine the influence of their direction. Hereafter, each "v" and "h" identifies a vertical or horizontal type.

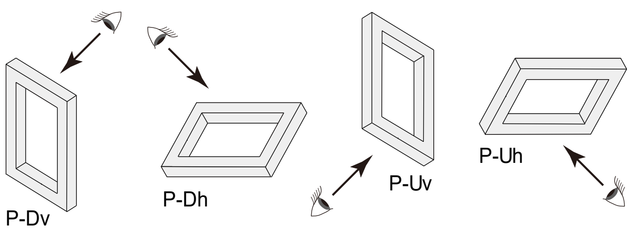


Figure 8 Possible rectangles

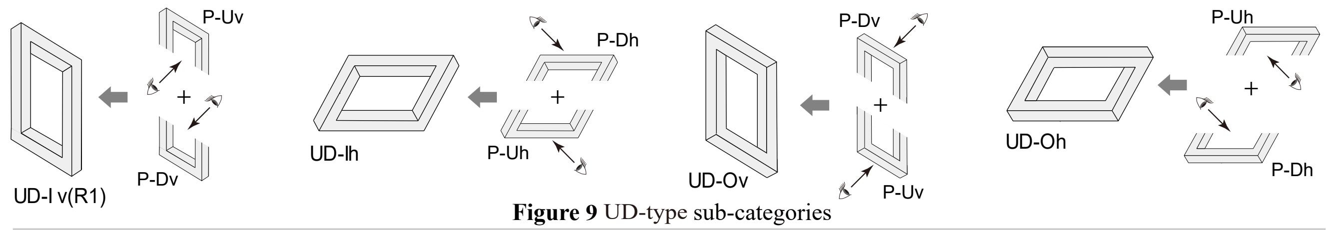


Figure 9 UD-type sub-categories

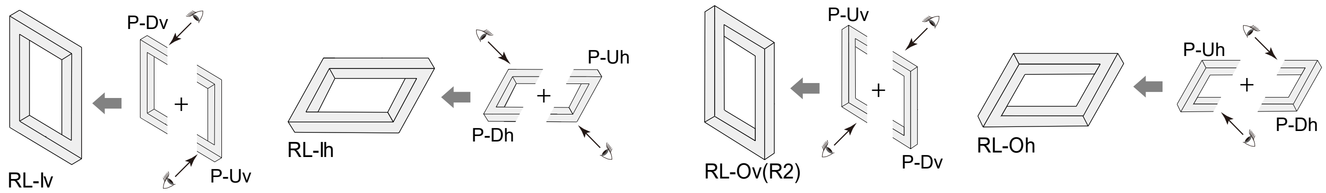
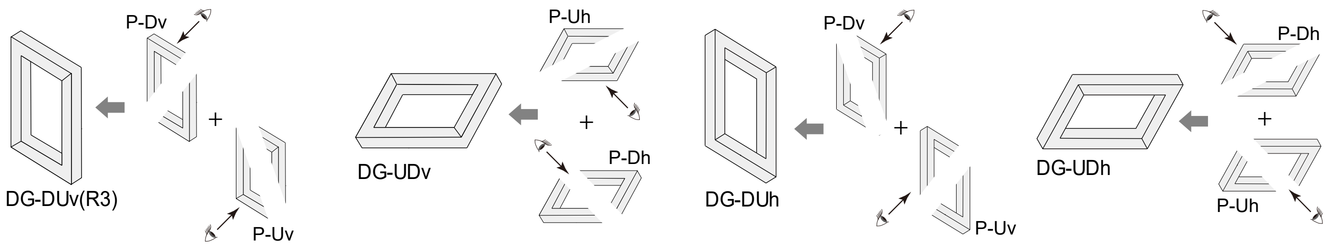


Figure 10 RL-type sub-categories



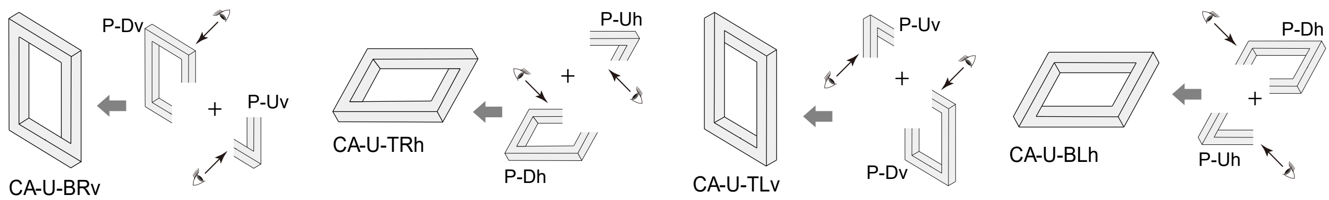
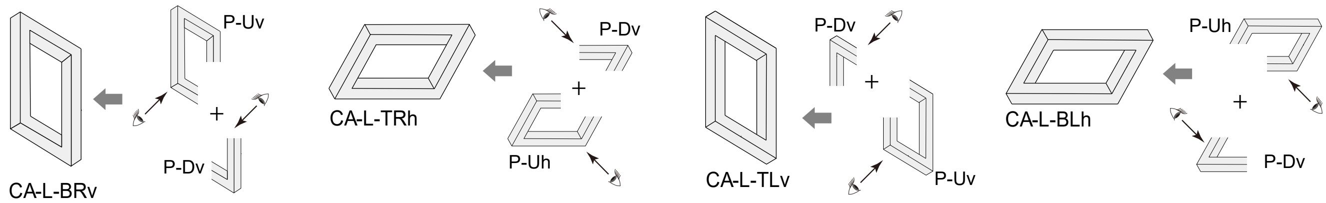


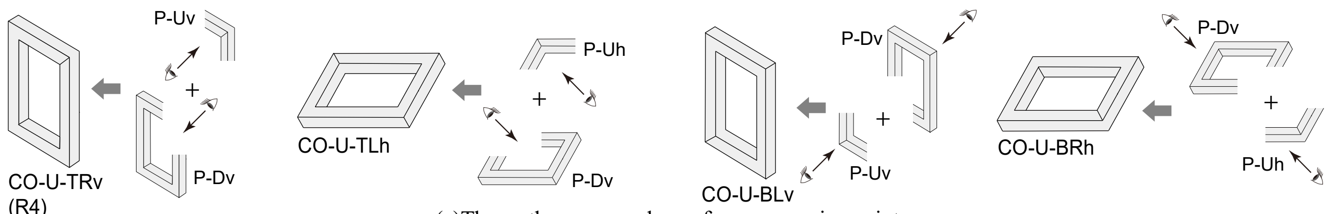
Figure 11 DG-type sub-categories

(a)Three other corners drawn from upper viewpoint

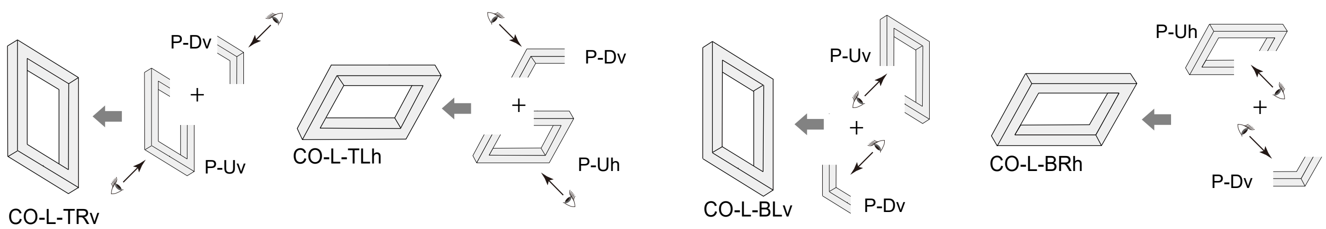


(b)Three other corners drawn from lower viewpoint

Figure 12 CA-type sub-categories (One acute corner drawn from a different viewpoint)

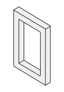
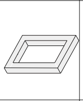
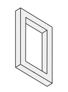
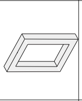
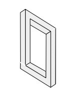
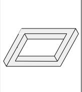
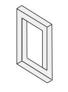
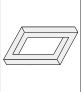
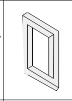
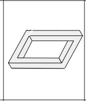
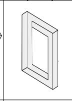
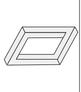
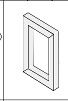
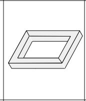
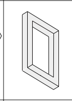
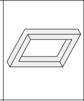


(a)Three other corners drawn from upper viewpoint



(b)Three other corners drawn from lower viewpoint

Figure 13 CO-type sub-categories (One obtuse corner drawn from a different viewpoint)

Attribute	Possible(P-type)				Upper and Lower (UD-type)				Right and Left (RL-type)				Diagonal (DG-type)			
Symbol	P-Dv	P-Dh	P-Uv	P-Uh	UD-Iv (R1)	UD-Ih	UD-Ov	UD-Oh	RL-Iv	RL-Ih	RL-Ov (R2)	RL-Oh	DG-Duv (R3)	DG-Duh	DG-Udv	DG-Udh
Figure																
Possible	100.0%	97.5%	100.0%	100.0%	7.5%	5.0%	2.5%	7.5%	5.0%	5.0%	0.0%	5.0%	30.0%	40.0%	37.5%	42.5%
Impossible	0.0%	2.5%	0.0%	0.0%	92.5%	95.0%	97.5%	92.5%	95.0%	95.0%	100.0%	95.0%	65.0%	60.0%	62.5%	55.0%
Unable to decide	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.5%

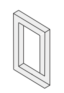
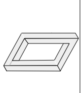
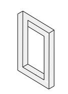
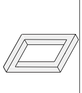
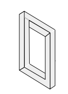
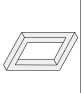
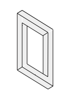
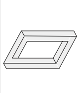
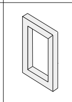
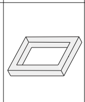
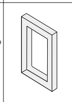
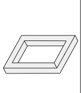
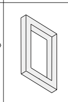
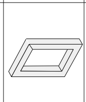
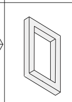
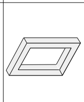
Attribute	One corner drawn from a different viewpoint (C-type)															
	One acute corner drawn from a different viewpoint (CA-type)								One obtuse corner drawn from a different viewpoint (CO-type)							
	Three other corners drawn from upper viewpoint				Three other corners drawn from lower viewpoint				Three other corners drawn from upper viewpoint				Three other corners drawn from lower viewpoint			
Symbol	CA-U -BRv	CA-U -TRh	CA-U -TLv	CA-U -BLh	CA-L -BRv	CA-L -TRh	CA-L -TLv	CA-L -BLh	CO-U- TRv(R4)	CO-U -TLh	CO-U -BLv	CO-U -BRh	CO-L -TRv	CO-L -TLh	CO-L -BLv	CO-L -BRh
Figure																
Possible	7.5%	0.0%	7.5%	0.0%	0.0%	5.0%	2.5%	0.0%	40.0%	42.5%	32.5%	45.0%	42.5%	42.5%	37.5%	37.5%
Impossible	92.5%	100.0%	92.5%	100.0%	100.0%	95.0%	97.5%	100.0%	60.0%	57.5%	65.0%	55.0%	57.5%	57.5%	60.0%	62.5%
Unable to decide	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.5%	0.0%	0.0%	0.0%	2.5%	0.0%

Table 2 Result of Experiment 2

Figure 9 shows the four sub-categories of the UD-type. UD-Iv includes the upper side of P-Uv and the lower side of P-Dv. In other words, the upper side of UD-Iv is viewed from below and the lower side is viewed from above. In contrast, UD-Ov includes the upper side of P-Dv and the lower side of P-Uv; thus, the upper side of UD-Ov is viewed from above and the lower side is viewed from below. UD-lh similarly comprises the upper side of P-Dh and the lower side of P-Uh, and UD-Oh includes the upper side of P-Uh and the lower side of P-Dh.

Figure 10 shows the four sub-divisions of the RL-type. RL-Iv includes the right side of P-Dv and the left side of P-Uv. Each RL-Ih, RL-Ov, and RL-Oh similarly has a structure as shown in Figure 10.

In Figure 11, the four categories of the DG-type are shown. The top left and bottom right corner pairs of DG-DUv include P-Dv, and the top right and bottom left corner pairs include P-Uv. DG-Duh, DG-UDv, and DG-UDh similarly have structures as P-Uv shown in Figure 11, respectively.

C-type is further sub-classified into two groups. One group has a single acute corner drawn from a different viewpoint and is termed CA-type in Figure 12. The other group has one obtuse corner drawn from a different viewpoint and is termed CO-type in Figure 13. Further, CA-type and CO-type are divided by view position. One sub-group includes three other corners which are drawn from upper viewpoint shown in Figures 12(a) and 13(a). The other sub-group includes three other corners which are drawn from lower viewpoint shown in Figures 12(b) and 13(b). In the case of CA-U-BRv, in which the bottom right corner includes P-Uv as viewed from below while the other part includes P-Dv as viewed from above. Other figures that belong to C-type have structures as shown in Figures 12 and 13.

4-3 Experiment 2

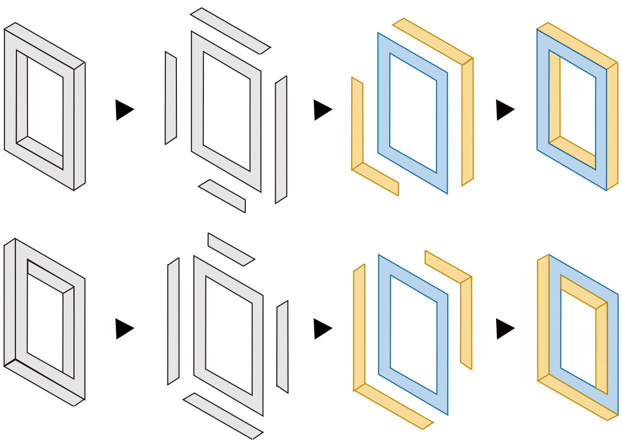
Experiment 2 was conducted with 40 participants (25 male, 15 female, average age 22 years). Five of them also participated in Experiment 1, while the remaining 35 did not know anything about Experiment 1. The figures were drawn using black 1.5pt lines, and every polygon was painted in monochrome with 90% brightness. The other experimental conditions were the same as those of Experiment 1

4-4 Result

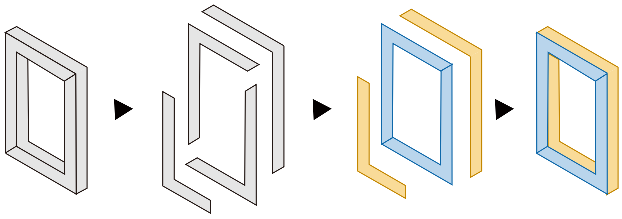
The results of Experiment 2 are shown in Table 2. UD-, RL-, and DG-type in Experiment 2 did not differ from those in Experiment 1. Therefore, hypotheses 1, 2, and 3 were confirmed; however, the result of C-type was different between CA-type and CO-type. The figures in CA-type were mostly perceived as impossible figures. In contrast, approximately 40% of the participants perceived CO-type as possible figures. Thus, hypothesis 4 was only partially supported.

5 Discussion

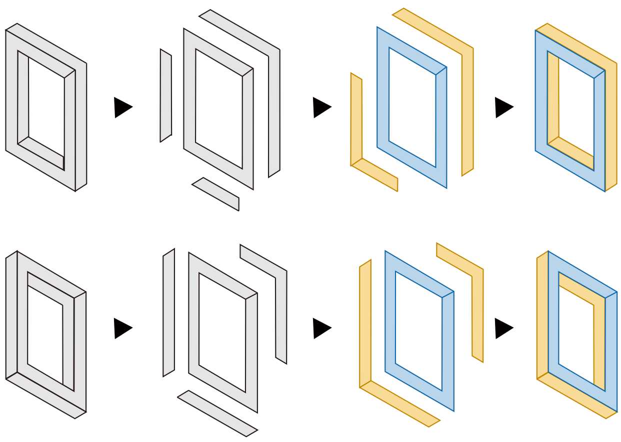
To examine the results of Experiment 2 in more detail, we broke down every figure into plane polygons. We only showed one 2D-rotated figure. In Figure 14(a), each possible rectangle was broken down into four I-shaped polygons and a plane rectangle. L-shaped polygons in yellow were formed by jointing two I-shaped polygons. Thus, a possible rectangle could be viewed as comprising two L-shaped polygons in yellow and a plane rectangle in blue. The DG-type was divided into four L-shaped polygons, as shown in Figure 14(b). A plane rectangle in blue was formed by connecting two L-shaped polygons. The DG-type could also be thought of as comprising two yellow L-shaped polygons and a blue plane rectangle. Similarly, each CO-type could be viewed as comprising two yellow L-shaped polygons and a blue plane rectangle, as shown in Figure 14(c).



(a) Possible rectangle



(b) DG-type

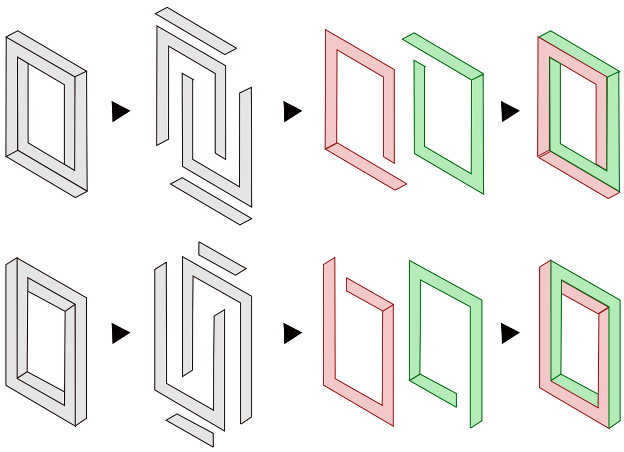


(c) CO-type

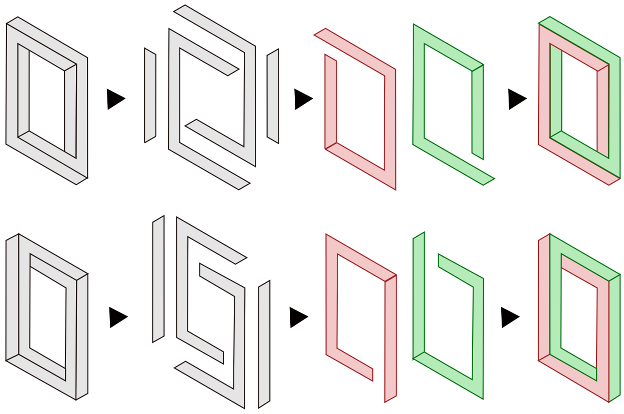
Figure 14 Element led to possible figures

Thus, the DG- and CO-types, which have a possibility not to be perceived as an impossible figure, have a similar polygon structure and share a possible rectangle. This blue rectangle is considered to be the cause for participants' difficulty in distinguishing between possible and impossible figures. Thus, this is one of the elements that leads to participants' perception of possible figures.

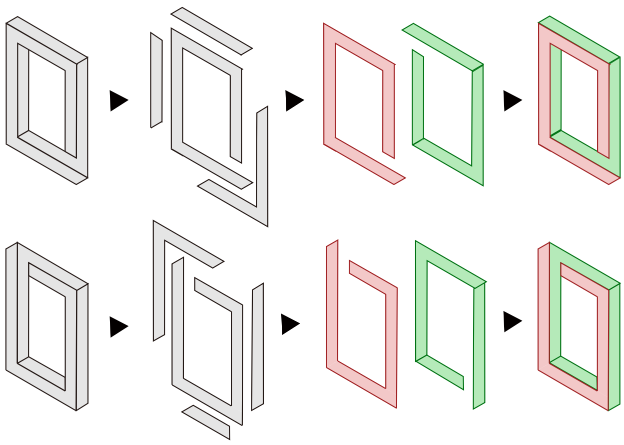
In contrast, each UD-type is broken down into two U-shaped and two I-shaped polygons, as shown in Figure 15(a). Pink and green open plane rectangles were formed by connecting a U-shaped polygon and an I-shaped polygon, respectively. Each UD-type could be considered to comprise two each of the pink and green open plane rectangles. Furthermore, the RL-type is



(a) UD-type



(b) RL-type



(c) CA-type

Figure 15 Elements perceived easily as impossible figures

identical, as shown in Figure 15(b). Each CA-type was divided into an L-shaped polygon, two I-shaped polygons, and an open plane rectangle, as shown in Figure 15(c). Similarly, the CA-type could be viewed as comprising two open plane rectangles in pink and green. The UD-type, RL-type, and CA-type, which have high chances of being perceived as impossible figures, can be viewed as comprising pink and green rectangles in common. These two open plane rectangles gave a feeling of torsion and are considered to be the elements leading to participants' perception of impossible figures.

6 Conclusion

Impossible figures have been examined in various fields; however, although they are mind images, the different perceptions of impossible figures have not been sufficiently investigated. In such a situation, through this study, we indicated that the perception of impossible figures differs according to viewers and the figures themselves, as established in Experiment 1. Furthermore, we also found the elements that led to viewers' perception of impossible and possible figures in Experiment 2, which focused on inconsistent rectangles having external contours of possible rectangles. To further contribute to future studies and creative works related to impossible figures, the analysis outlined in this study will be expanded to include general impossible figures.

Reference

[1]Seckel,A., *Masters of Deception*, Sterling Publishing Co.,Inc, 2004.

[2]Ernst, B. *Magic Mirror of M.C.Escher*, Taschen, 1978.

[3]Penrose L S, Penrose R, *Impossible objects :A special type of visual illusion*, British Journal of Psychology 49 pp.31-33,1958.

[4]Gregory, R. L., *The Intelligent Eye*, Weidenfeld and Nicolson, 1971

[5]Robinson, J. O, *The Psychology of Visual Illusion*, Hutchinson, 1972.

[6]Draper S.W., *The Penrose Triangle and a Family of Related Figures*, Perception vol.7 pp.283-296, 1978.

[7]Cowan T M, *The theory of braids and the analysis of impossible figures*, Journal of Mathematical Psychology, vol.11, pp.190-212, 1974.

[8]Cowan T.M., *Organizing the properties of impossible figures*, Perception vol. 6 pp.41-56 1977.

[9]Cowan T.M, Pringle R, *An investigation of the cues responsible for figure impossibility*, Journal of Experimental Psychology. Human Perception and Performance vol.4 pp.112-120 1978.

[10]Kulpa Z., *Are impossible figures possible*, Signal Processing vol.5 pp.201-220, 1983.

[11]Kupla Z., *Putting order in the impossible,* Peceptions vol.16 201-14, 1987.

[12]Gillam B., *Even a possible figure can look impossible*, Perception vol.8 pp.229-232 1979

[13]Young A W., Deregowski J B., *Learning to see impossible*, Perception vol.10 pp.91-105 1981.

[14] Shepard R.N., *Mind Sight*, W.H.Freeman and Company, New York, 1990

[15]Ernst B. Adventures with impossible figures. Tarquin Publications, 1986

[16]Sugihara K, *Machine Interpretation of Line Drawings*. MIT Press, Cambridge, 1986

[17]Sugihara, K. . *Three-dimensional realization of anomalous pictures-an application of picture interpretation theory to toy design*. Pattern Recognition vol.30,7, pp. 1061-1067. 1997

[18] T´erouanne E, *On a class of 'impossible' figures: a new language for a new analysis*, Journal of Mathematical Psychology vol.22 pp.24-46, 1980.

[19]Uribe D., *A set of impossible tiles*, The third international conference Mathematics and Design, Available at http://im-possible.info/english/articles/tiles/tiles.html, 2001.

[20]Huffman D. A., *Impossible objects as nonsense sentences*. Machine Intelligence 6 pp. 295-323, 1971.

[21] Clowes M. B. *On seeing things*. Artificial Intelligence, vol. 2, pp.79-116, 1971.

[22]Tsuruno S., *The animation of M.C. Escher's 'Belvedere'*, ACM SIGGRAPH 97 Visual Proceeding, p237. Presented at Siggraph Electronic Theater, 1997.

[23]Tsuruno, S. *Natural Expression of Physical Models of Impossible Figures and Motions*, International Journal of Asia Digital Art and Design Vol.18, No.04, pp88-95,2015.

[24]Savransky G., Dimermanz D., Gotsman C., *Modeling and Rendering Escher-Like Impossible Scenes*, Computer Graphics forum, Vol.18, no.2, pp.173-179, 1999.

[25]Owada S., Fujiki J., *Dynafusion: A modeling system for interactive impossible objects*, In Proc. of Non-Photorealistic Animation and Rendering (NPAR), pp. 65-68, 2008.

[26]Wu T.-P., Fu C.-W., Yeung S.-K., Jia J., Tang C.-K., *Modeling and rendering of impossible figures*, ACM Transactions on Graphics, Vol. 29, No. 4, Article 106, 2010.

[27]Elber, G. *Modeling (seemingly) impossible models*, Computers and Graphics35, pp632-638, 2011.

[28]Yturralde J M, *"Ambigous structures" in Hypergraphics - Visualizing Complex Relationships in Art,* Science and Technology ed D W Brisson (Boulder: CO Westview Press) pp. 177-185,1978

[29]Sugihara, K., *Impossible motion: magnet-like slopes*, First Prize, in the 2010 Best Illusion Contest of the Year, 2010.

[30]Tsuruno S., *Illusion of Height Contradiction* , Top 10 finalists in the 2012 Best Illusion Contest of the Year, 2012.

[31]Cole F., Sanik K.,DeCarlo D., Finkelstein A., Funkhouser T., Rusinkiewicz S., Singh M., *How Well Do Line Drawings Depict Shape?* ACM Transactions on Graphics, Vol. 28, No. 3, Article 28, 2009.

[32]Lee Y., Kimy Y., Kangz H., Kangz.H., *Binocular Depth Perception of Stereoscopic 3D Line Drawings*, SAP '13 Proceedings of the ACM Symposium on Applied Perception pp. 31-34,2013